Simplification of the Gaussian Integral

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1 Introduction

The Gaussian integral is known as the probability integral and closely related to the error function [1] It has great impact in the field of science and engineering, from the applications of chemical engineering to statistics. This integral is famous for its solving complexity and one must be able to manipulate mathematical techniques to obtain the final result. To add on, this is my first latex program and I would like to also thank Youtube and ChatGPT for making this more simple to complete.

2 Clarification

Despite my status as an undergraduate student at the University of Michigan, I have to state my intentions of writing this paper out of purely personal interest and this paper is not affiliated to any of the departments in the University of Michigan.

3 Solution

3.1 Cartesian coordinates

$$\int_{-\infty}^{\infty} e^{-x^2} = \sqrt{\pi}$$

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$$I = \int_{-\infty}^{\infty} e^{-x^2}\, dx$$

$$I^2 = \int_{-\infty}^{\infty} e^{-x^2}\, dx \int_{-\infty}^{\infty} e^{-x^2}\, dx$$

$$I^2 = \int_{-\infty}^{\infty} e^{-x^2}\, dx \int_{-\infty}^{\infty} e^{-y^2}\, dy$$

$$I^2 = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} e^{-x^2} e^{-y^2}\, dx\, dy$$

3.2 Convert to Polar coordinates

$$x = r\cos(\theta),$$
$$y = r\sin(\theta),$$
$$dx\,dy = r\,dr\,d\theta,$$
$$x^2 + y^2 = r^2.$$

$$I^2 = \int_0^{2\pi} \int_0^{\infty} e^{-(x^2+y^2)} r\,dr\,d\theta$$

$$I^2 = \int_0^{2\pi} \int_0^{\infty} e^{-r^2} r\,dr\,d\theta$$

$$I^2 = \int_0^{2\pi} \left(\int_0^{\infty} re^{-r^2}\,dr \right) d\theta$$

$$I^2 = \int_0^{2\pi} \left(-\frac{1}{2} e^{-r^2} \Big|_0^{\infty} \right) d\theta$$

$$I^2 = \int_0^{2\pi} \frac{1}{2}\,d\theta$$

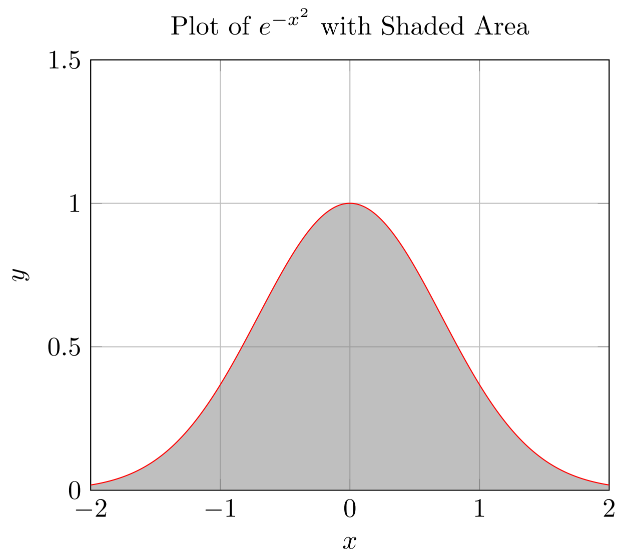
$$I^2 = \frac{1}{2}\theta \Big|_0^{2\pi}$$

$$I^2 = \pi$$

$$I = \sqrt{\pi}$$

4 Graph

Here is a graph of the Gaussian function e^{-x^2} with the entire visible area shaded:



5 Conclusion

The Gaussian Integral is represented as $\sqrt{\pi}$. The ability to manipulate mathematical methods to find a precise solution gives the Gaussian integral the ability to be seen as an important entity in the field of science and engineering. We are able to estimate that from the graph, if we assume that it is not a curve but triangles, we can estimate an area of 2 units squared from $x = -2$ to $x = 2$. However, if we finally choose to take in the bell curve nature of the graph and the range from $x = -\infty$ to $x = \infty$, we are able to see that it would be $\sqrt{\pi}$. This can also be supported by the fact that 2 units squared is not far from $\sqrt{\pi}$, which is approximately 1.772 rounded to three significant figures.

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References

[1] Eric W Weisstein. Gaussian integral. *https://mathworld. wolfram. com/*, 2004.